A note on ampleness in the theory of non abelian free groups

Rizos Sklinos

Abstract

Recently Ould Houcine-Tent [OHT12] proved that the theory of non abelian free groups is n-ample for any $n < \omega$. We give a sequence of primitive elements in $\mathbb{F}_\omega$ witnessing the above mentioned result. Our proof is not independent from [OHT12] as we essentially use some theorems from there. On the other hand our witnessing sequence is much simpler.

1 Introduction

The notion of n-ampleness, for some natural number n, fits in the general context of geometric stability theory. As the definition may look artificial or technical, we first give the historical background of its development. We start by working in a vector space V and we consider two finite dimensional subspaces $V_1, V_2 \leq V$. Then one can see that $dim(V_1 + V_2) = dim(V_1) + dim(V_2) - dim(V_1 \cap V_2)$, and the point really is that V_1 is linearly independent from V_2 over $V_1 \cap V_2$. In an abstract stable theory the notion of linear independence is replaced by forking independence and the above property gives rise to the notion of 1-basedness. A stable theory T is 1-*based* if there are no a, b such that $acl^{eq}(a) \cap acl^{eq}(b) = acl^{eq}(\emptyset)$ and a forks with b over $\emptyset$. The notion of 1-basedness turned out to be very fruitful in model theory and one of the major results concerning this notion was the following theorem by Hrushovski-Pillay [HP85].

Theorem 1.1: *Let $\mathcal{G}$ be a 1-based stable group. Then every definable set $X \subseteq G^n$ is a Boolean combination of cosets of almost $\emptyset$-definable subgroups of G^n. Moreover G is abelian-by-finite.*

On the other hand, Hrushovski's seminal work in refuting Zilber's trichotomy conjecture (see [Hru93]) produced "new" strongly minimal sets that had an interesting property. Hrushovski isolated this property and called it CM-triviality (for Cohen-Macaulay). A stable theory T is CM-*trivial* if there are no a, b, c such that a forks with c over $\emptyset$, a is independent from c over b, $acl^{eq}(a) \cap acl^{eq}(b) = acl^{eq}(\emptyset)$ and finally $acl^{eq}(a, b) \cap acl^{eq}(a, c) = acl^{eq}(a)$. A kind of an analogue to the moreover statement of the above theorem has been proved by Pillay in [Pil95].

Theorem 1.2: *A CM-trivial group of finite Morley rank is nilpotent-by-finite.*

Pillay first realized the pattern and proposed an hierarchy of ampleness, non 1-basedness (1-ampleness) and non CM-triviality (2-ampleness) being the first two items in it (see [Pil00]). His definition needed some fine "tuning" as observed by Evans [Eva03].

Definition 1.3 ([Eva03])**:** *Let T be a stable theory and $n \geq 1$. Then T is n-ample if (after possibly adding some parameters) there are $a_0, a_1, \ldots, a_n$ such that:*

1. *a_0 forks with a_n over $\emptyset$;*

2. a_{i+1} *does not fork with* $a_0, \ldots, a_{i-1}$ *over* a_i, *for* $1 \leq i < n$;

3. $acl^{eq}(a_0) \cap acl^{eq}(a_1) = acl^{eq}(\emptyset)$;

4. $acl^{eq}(a_0, \ldots, a_{i-1}, a_i) \cap acl^{eq}(a_0, \ldots, a_{i-1}, a_{i+1}) = acl^{eq}(a_0, \ldots, a_{i-1})$, *for* $1 \leq i < n$.

The purpose of this paper is to give an alternative sequence to the one given in [OHT12] witnessing n-ampleness, for any $n < \omega$, in the theory of non abelian free groups. The advantage of our sequence is that it is much simpler and consists only of primitive elements (instead of triples of elements). Though the witnessing sequence is the only point that we diverge from [OHT12] we try to make the paper self-contained.

The paper is organized as follows. In section 2, we give some background around the main geometric tool, i.e. JSJ-decompositions.

In section 3 we analyze the three main ingredients of the proof, i.e. the geometric elimination of imaginaries [Sel], the understanding of algebraic closure [OHV11, OHT12], and finally the understanding of forking independence for primitive elements [Pil08, Pil09], in non abelian free groups.

In the final section we give explicitly a sequence witnessing n-ampleness for any $n < \omega$.

Our notation is standard. By $\mathbb{F}_n$ we denote the free group of rank n, and by T_{fg} the common theory of non abelian free groups. If $\mathbb{M}$ is a "big" saturated model of a first order theory T and $A \subset \mathbb{M}$, then by $acl_{\mathbb{M}}(A) = acl(A)$ we mean the "real" algebraic closure, while by $acl^{eq}_{\mathbb{M}}(A) = acl^{eq}(A)$ we denote the algebraic closure computed in T^{eq}.

2 JSJ decompositions

In this section we will describe the notion of JSJ-decompositions. Roughly speaking, a JSJ-decomposition of a group G is a splitting as a graph of groups in which one can "read" all possible splittings of G over a given class, $\mathcal{A}$, of subgroups, i.e. splittings of G where all edge groups belong to the class $\mathcal{A}$. Note that it is not immediate that such a splitting exists (provided $\mathcal{A}$ is given). Actually the existence of (cyclic) JSJ-decompositions for hyperbolic groups [Sel97] by Sela and later for finitely presented groups [RS97] by Rips-Sela was a major breakthrough in group theory. After that, various results appeared mainly extending the class of subgroups $\mathcal{A}$, the more general being Fujiwara-Papazoglu [FP06], extending Rips-Sela by taking $\mathcal{A}$ to be the class of slender subgroups, i.e. groups for which all subgroups are finitely generated.

We give a more formal account of the JSJ theory following the unifying framework of Guirardel-Levitt (see [GL09a], [GL09b]). We note that we will change our point of view from group splittings to groups acting on trees and the other way around (using the duality explained by Bass-Serre theory [Ser83]) when it is convenient, but it will always be clear what we mean from the context.

Let G be a group acting on a tree T (by automorphisms and without inversions), we call T a *cyclic G-tree* if all edge stabilizers are cyclic. If H is a subgroup of G, then H is *elliptic* in T if it fixes a point in T, otherwise it is called *hyperbolic*. We fix a group G and we work in the class, $\mathcal{T}_G$, of all cyclic G-trees. A tree in $\mathcal{T}_G$ is *universally elliptic* if its edge stabilizers are elliptic in every tree in $\mathcal{T}_G$. If T_1, T_2 are two trees in $\mathcal{T}_G$, we say that T_1 *dominates* T_2 if every subgroup of G which is elliptic in T_1, is elliptic in T_2.

A *cyclic JSJ-tree* is a universally elliptic tree which dominates any universally elliptic tree. We will be also interested in relative cyclic JSJ-trees of a group G with respect to a family of subgroups $\mathcal{H}$. In this case we work in the class of all cyclic G-trees in which every $H \in \mathcal{H}$ is elliptic. Finally, by a *(relative) cyclic JSJ-decomposition* of a group G we

mean the corresponding graph of groups obtained by the action of G on a (relative) cyclic JSJ-tree.

Let G be a group and $\mathcal{H}$ be a family of subgroups of G, let $\mathcal{T}_{(G,\mathcal{H})}$ be the class of all cyclic G-trees in which every $H \in \mathcal{H}$ is elliptic. Let T_{JSJ} be a relative cyclic JSJ-tree in $\mathcal{T}_{(G,\mathcal{H})}$. A vertex in T_{JSJ} is called *rigid* if the vertex stabilizer is elliptic in any other tree in $\mathcal{T}_{(G,\mathcal{H})}$, and *flexible* if not.

In this context the essential part of the JSJ-theory is the description of the flexible vertex stabilizers of a JSJ-tree (provided such a tree exists). We give the description in the case of torsion-free hyperbolic groups.

We first recall that the fundamental group of a compact surface, Σ, with boundary is a free group. Each boundary component of Σ has cyclic fundamental group, and gives rise in $\pi_1(\Sigma)$ to a conjugacy class of cyclic subgroups: we call these *maximal boundary subgroups.* A *boundary subgroup* of $\pi_1(\Sigma)$ is subgroup of a maximal boundary subgroup of $\pi_1(\Sigma)$.

Now, let G_u be a vertex stabilizer in a tree which is in $\mathcal{T}_{(G,\mathcal{H})}$. Then G_u is *quadratically hanging* if it is the fundamental group of a surface Σ with boundary, each incident edge group is a boundary subgroup and every conjugate of a group in $\mathcal{H}$ intersects G_u in a boundary subgroup. In this case, we say that a boundary component C of the surface Σ is *used* if there exists an incident edge group or a subgroup of G_u conjugate to some $H \in \mathcal{H}$ that is contained in $\pi_1(C)$ as a finite index subgroup.

The following theorem is essentially due to Sela (non-relative case), but the relative case is contained in [GL09a, Theorem 8.20].

Theorem 2.1: *Let G be a torsion-free hyperbolic group freely indecomposable with respect to a subgroup H. Then a cyclic relative JSJ-decomposition exists. The flexible vertex groups are quadratically hanging and every component of the corresponding surfaces is used.*

Note that in this case quadratically hanging vertex groups are also called vertex groups of *surface type.*

The following lemma will be useful in practice.

Lemma 2.2: *[GL09a, Lemma 5.3] Let G be finitely generated. Let T be a universally elliptic G-tree. Then G has a JSJ-tree if and only if every vertex stabilizer G_u of T has a relative JSJ-tree with respect to P_u (the set of incident edge stabilizers). Moreover, the JSJ-tree is obtained by refining T using these trees.*

Remark 2.3: *[GL09a, Remark 5.4] Lemma 2.2 is true in the relative case (where $\mathcal{H}$ is the family of "fixed" subgroups of G), provided that one adds to P_u all subgroups of G_u that are conjugate to some $H \in \mathcal{H}$.*

We give some easy examples of relative cyclic JSJ-decompositions.

Example 2.4: (i) The relative JSJ-decomposition of $\mathbb{F}_2 = \langle e_1, e_2 \rangle$ with respect to $\langle [e_1, e_2] \rangle$ is the following: $\mathbb{F}_2 *_{\langle [e_1,e_2] \rangle} \langle [e_1, e_2] \rangle$.

(ii) The relative JSJ-decomposition of $\mathbb{F}_4 = \langle e_1, e_2, e_3, e_4 \rangle$ with respect to $\langle [e_1, e_2][e_3, e_4] \rangle$ is the following: $\mathbb{F}_4 *_{\langle [e_1,e_2][e_3,e_4] \rangle} \langle [e_1, e_2][e_3, e_4] \rangle$.

(iii) More generally, the relative JSJ-decomposition of $\mathbb{F}_{2n} = \langle e_1, \ldots, e_{2n} \rangle$ with respect to $\langle [e_1, e_2][e_3, e_4] \ldots [e_{2n-1}, e_{2n}] \rangle$ is the following: $\mathbb{F}_{2n} *_{\langle [e_1,e_2][e_3,e_4]\ldots[e_{2n-1},e_{2n}] \rangle} \langle [e_1, e_2] [e_3, e_4] \ldots [e_{2n-1}, e_{2n}] \rangle$.

The following pictures give the intuition behind the first two above mentioned examples.

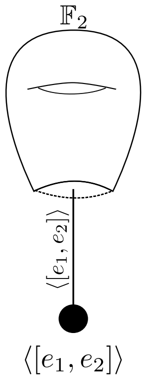

Figure 1: The relative JSJ of $\mathbb{F}_2$ with respect to $\langle [e_1, e_2] \rangle$

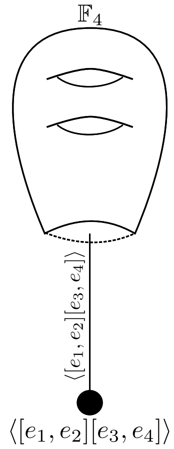

Figure 2: The relative JSJ of $\mathbb{F}_4$ with respect to $\langle [e_1, e_2][e_3, e_4] \rangle$

3 Imaginaries, Algebraic closure, Forking

The main result that allowed the proof in [OHT12] is the geometric elimination of imaginaries due to Sela [Sel]. Although in this paper we do not use it directly, we state it for completeness. We start by defining some "tame" families of imaginaries.

Definition 3.1: *Let G be a torsion-free hyperbolic group. The following equivalence relations in G are called basic.*

(i) $E_1(a, b)$ *if and only if there is $g \in G$ such that $a^g = b$. (conjugation)*

(ii)$_m$ $E_{2_m}((a_1, b_1), (a_2, b_2))$ *if and only if* $b_1, b_2 \neq 1$ *and* $C_G(b_1) = C_G(b_2) = \langle b \rangle$ *and* $a_1^{-1} a_2 \in \langle b^m \rangle$. *(m-left-coset)*

(iii)$_m$ $E_{3_m}((a_1, b_1), (a_2, b_2))$ *if and only if* $b_1, b_2 \neq 1$ *and* $C_G(b_1) = C_G(b_2) = \langle b \rangle$ *and* $a_1 a_2^{-1} \in \langle b^m \rangle$. *(m-right-coset)*

(iv)$_{m,n}$ $E_{4_{m,n}}((a_1, b_1, c_1), (a_2, b_2, c_2))$ *if and only if* $a_1, a_2, c_1, c_2 \neq 1$ *and* $C_G(a_1) = C_G(a_2) = \langle a \rangle$ *and* $C_G(c_1) = C_G(c_2) = \langle c \rangle$ *and there is* $\gamma \in \langle a^n \rangle$ *and* $\epsilon \in \langle c^n \rangle$ *such that* $\gamma b_1 \epsilon = b_2$. *($m, n$-double-coset)*

Sela proved geometric elimination of imaginaries up to the basic sorts (see [Sel]).

Theorem 3.2: *Let G be a torsion-free hyperbolic group. Let $E(\bar{x}, \bar{y})$ be a $\emptyset$-definable equivalence relation in G, with $|\bar{x}| = m$. Then there exist $k, l < \omega$ and a $\emptyset$-definable relation:*

$$R_E \subseteq G^m \times G^k \times S_1(G) \times \ldots \times S_l(G)$$

such that:

(i) *each $S_i(G)$ is one of the basic sorts;*

(ii) *for each $\bar{a} \in G^m$, $|R_E(\bar{a}, \bar{z})|$ is uniformly bounded (i.e. the bound does not depend on $\bar{a}$);*

(iii) $R_E(\bar{a}, \bar{z}) \leftrightarrow R_E(\bar{b}, \bar{z})$ *if and only if* $E(\bar{a}, \bar{b})$.

The following theorem of Ould Houcine-Vallino gives an understanding of the algebraic closure in free groups with respect to JSJ-decompositions (see [OHV11]). We recall that given an abelian splitting of G, i.e. a splitting in which all edge groups are abelian, then the *elliptic abelian neighborhood* of a vertex group in this splitting is the subgroup generated by the elliptic elements that commute with nontrivial elements of the vertex group.

Theorem 3.3: *Let A be a non abelian subgroup of $\mathbb{F}_n$, such that $\mathbb{F}_n$ is freely indecomposable with respect to A. Then $acl(A)$ coincides with the elliptic abelian neighborhood of the vertex group containing A in the cyclic JSJ-decomposition of $\mathbb{F}_n$ relative to A.*

For the purpose of this paper the following theorems of Pillay concerning forking independence and genericity are enough.

Theorem 3.4 (Corollary 2.7(ii)[Pil08])**:** *Any basis of $\mathbb{F}_n$ is an independent set.*

Theorem 3.5 (Theorem 2.1(i)[Pil09])**:** *Suppose a is a generic element in $\mathbb{F}_n$. Then a is primitive.*

For completeness we give the description of forking independence over free factors given by Perin-Sklinos [PS].

Theorem 3.6: *Let $\bar{a}, \bar{b} \in \mathbb{F}_n$ and G be a free factor (possibly trivial) of $\mathbb{F}_n$. Then $\bar{a}$ does not fork with $\bar{b}$ over G if and only if $\mathbb{F}_n$ admits a free decomposition $\mathbb{F}_n = \mathbb{F} * G * \mathbb{F}'$ and $\bar{a} \in \mathbb{F} * G$ and $\bar{b} \in G * \mathbb{F}'$.*

We will also use the following theorems from [OHT12]. We note that $acl^c(A)$ denotes the set of conjugacy classes in $acl^{eq}(A)$.

Theorem 3.7: *Let $\bar{a}, \bar{b}, \bar{g}$ be finite tuples from $\mathbb{F}_n$. Then we have:*

$$acl^{eq}(\bar{a}) \cap acl^{eq}(\bar{b}) = acl^{eq}(\bar{g})$$

if and only if

$$acl(\bar{a}) \cap acl(\bar{b}) = acl(\bar{g})$$

and

$$acl^c(\bar{a}) \cap acl^c(\bar{b}) = acl^c(\bar{g})$$

Theorem 3.8: *Let A be a non abelian subgroup of $\mathbb{F}_n$, such that $\mathbb{F}_n$ is freely indecomposable with respect to A. Let $b \in \mathbb{F}_n$. Then the conjugacy class of b belongs to $acl^c(A)$ if and only if in any cyclic JSJ-decomposition of $\mathbb{F}_n$ relative to A, either b is conjugate to some element of the elliptic abelian neighborhood of a rigid vertex group or it is a conjugate to an element of a boundary subgroup of a surface type vertex group.*

4 Witnessing Ampleness

The following sequence in $\mathbb{F}_\omega$ witnesses n-ampleness, for any $n < \omega$. We give the sequence recursively:

$$a_0 = e_1$$

$$a_{i+1} = a_i[e_{2i+2}, e_{2i+3}], \text{ for } 0 \leq i < \omega$$

We fix a natural number $n \geq 1$, and we show that $a_0, \ldots, a_n$ witnesses n-ampleness by verifying the requirements of Definition 1.3.

Lemma 4.1: *$a_0 = e_1$ forks with $a_n = e_1[e_2, e_3] \ldots [e_{2n}, e_{2n+1}]$.*

Proof. Immediate, since $[e_2, e_3] \dots [e_{2n}, e_{2n+1}]$ is not a primitive element, thus by Theorem 3.5 is not generic. $\square$

Lemma 4.2: *Let $1 \leq i < n$. Then $a_0, \dots, a_{i-1}$ does not fork with a_{i+1} over a_i.*

Proof. We first note that for each i, $\langle a_i \rangle$ is a free factor of $\mathbb{F}_{2i+3}$. Thus, by Theorem 3.4, we only need to find a free factorization $\mathbb{F}_{2i+3} = \mathbb{F} * \langle a_i \rangle * \mathbb{F}'$, such that $a_0, \dots, a_{i-1}$ is in $\mathbb{F} * \langle a_i \rangle$ and a_{i+1} is in $\langle a_i \rangle * \mathbb{F}'$. It is easy to see that the following free factorization is such $\mathbb{F}_{2i+3} = \langle e_2, \dots, e_{2i}, e_{2i+1} \rangle * \langle a_i \rangle * \langle e_{2i+2}, e_{2i+3} \rangle$. $\square$

Lemma 4.3: $acl^{eq}(e_1) \cap acl^{eq}(e_1[e_2, e_3]) = acl^{eq}(\emptyset)$.

Proof. It is not hard to see that $acl(e_1) = \langle e_1 \rangle$ and $acl^c(e_1)$ is the set of conjugacy classes of elements in $\langle e_1 \rangle$. The same is true for $acl(e_1[e_2, e_3])$ and $acl^c(e_1[e_2, e_3])$. Thus, $acl(e_1) \cap acl(e_1[e_2, e_3]) = acl(\emptyset)$ and $acl^c(e_1) \cap acl^c(e_1[e_2, e_3]) = acl^c(\emptyset)$. By Theorem 3.7 we are done.

$\square$

To verify Definition 1.3 (4) we first compute the JSJ-decomposition of $\mathbb{F}_{2i+1}$ relative to $\langle a_0, \dots, a_{i-1}, a_i \rangle$ and the JSJ-decomposition of $\mathbb{F}_{2i+3}$ relative to $\langle a_0, \dots, a_{i-1}, a_{i+1} \rangle$. We give the following pictures:

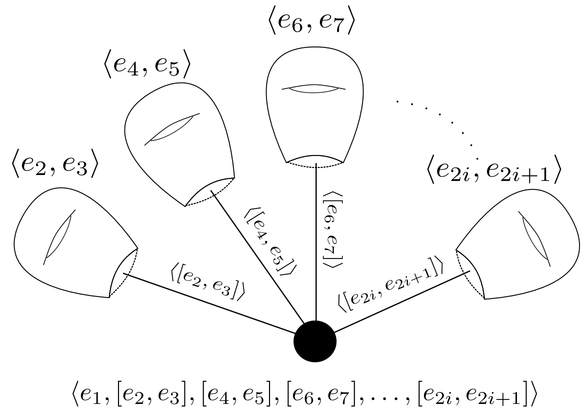

Figure 3: The relative JSJ-decomposition of $\mathbb{F}_{2i+1}$ with respect to $\langle a_0, \dots, a_{i-1}, a_i \rangle$

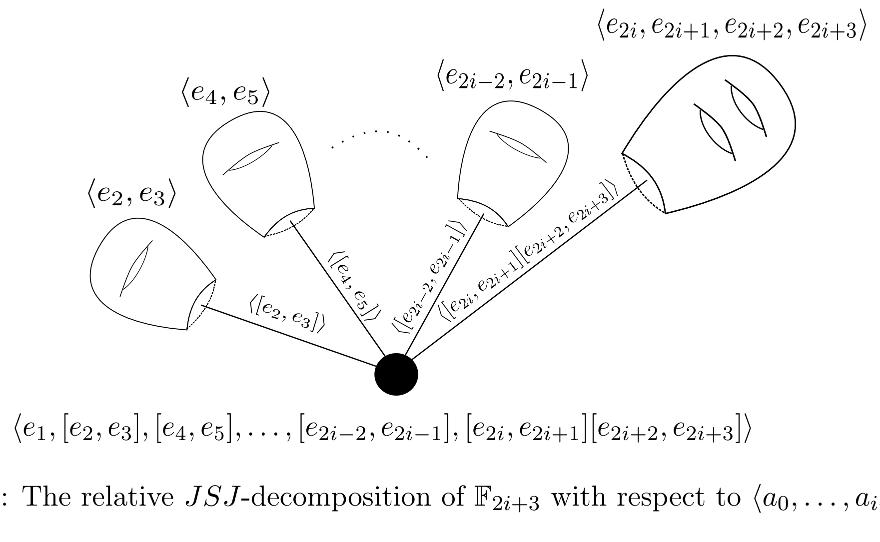

Figure 4: The relative JSJ-decomposition of $\mathbb{F}_{2i+3}$ with respect to $\langle a_0, \dots, a_{i-1}, a_{i+1} \rangle$

Lemma 4.4: *The JSJ-decomposition of $\mathbb{F}_{2i+1}$ relative to $\langle a_0, \ldots, a_{i-1}, a_i\rangle$ is the graph of groups given by figure* 3.

Proof. It is immediate that the given splitting is universally elliptic. That is a JSJ-decomposition follows from Remark 2.3 and Example 2.4 (i). $\square$

Lemma 4.5: *The JSJ-decomposition of $\mathbb{F}_{2i+3}$ relative to $\langle a_0, \ldots, a_{i-1}, a_{i+1}\rangle$ is the graph of groups given by figure* 4.

Proof. It is immediate that the given splitting is universally elliptic. That is a JSJ-decomposition follows from Remark 2.3 and Example 2.4 (i) and (ii). $\square$

We are now ready to finish our proof.

Lemma 4.6: *Let $1 \leq i < n$. Then $acl^{eq}(a_0, \ldots, a_{i-1}, a_i) \cap acl^{eq}(a_0, \ldots, a_{i-1},\ a_{i+1}) = acl^{eq}(a_0, \ldots, a_{i-1})$.*

Proof. We first note that by Theorem 3.3 and Lemmata 4.4,4.5 we have that $acl(a_0, \ldots, a_{i-1}, a_i) = \langle a_0, \ldots, a_{i-1}, a_i\rangle$ and $acl(a_0, \ldots, a_{i-1}, a_{i+1}) = \langle a_0, \ldots, a_{i-1}, a_{i+1}\rangle$. Thus, their intersection is $\langle a_0, \ldots, a_{i-1}\rangle$ which is exactly $acl(a_0, \ldots, a_{i-1})$.

By Theorem 3.7 we only need to show that $acl^c(a_0, \ldots, a_{i-1}, a_i) \cap acl^c(a_0, \ldots, a_{i-1}, a_{i+1}) = acl^c(a_0, \ldots, a_{i-1})$. But, by Theorem 3.8 and Lemmata 4.4,4.5 we have that $acl^c(a_0, \ldots, a_{i-1}, a_i)$ is exactly the set of conjugacy classes of elements in $\langle a_0, \ldots, a_{i-1}, a_i\rangle$ and $acl^c(a_0, \ldots, a_{i-1}, a_{i+1})$ is exactly the set of conjugacy classes of elements in $\langle a_0, \ldots, a_{i-1}, a_{i+1}\rangle$. Thus, their intersection is the set of conjugacy classes of elements in $\langle a_0, \ldots, a_{i-1}\rangle$, which is exactly $acl^c(a_0, \ldots, a_{i-1})$. $\square$

Putting everything together, our Lemmata 4.1,4.2,4.3,4.6, show that our sequence $(a_i)_{i<\omega}$ witnesses n-ampleness in the theory of non abelian free groups for each $n < \omega$. And thus giving an alternative sequence to the one used in [OHT12].

Theorem 4.7: *T_{fg} is n-ample for each $n < \omega$.*

Remark 4.8: *Actually we have produced a family of sequences witnessing the n-ampleness of T_{fg} for each $n < \omega$ as instead of once punctured tori we could have used any other once punctured surface (with a few exceptions of some obvious small surfaces).*

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Hebrew University of Jerusalem,
Einstein Institute of Mathematics,
91904, Israel
rsklinos@math.huji.ac.il